slcg SECURITIES LITIGATION & CONSULTING GROUP

The Use of Leveraged Investments to Diversify a Concentrated Position[1]

By

Craig McCann, PhD, CFA and Dengpan Luo, PhD

Introduction

Brokerage firms recently recommended that investors holding a concentrated position in a single stock borrow and invest in a portfolio of additional stocks to reduce risk. For example, Merrill Lynch distributed the following[2]:

> **Five Strategies for Diversifying a Concentrated Position**
>
> …
>
> There are several strategies that may be appropriate for diversifying concentrated positions and reducing a portfolio's risk.
>
> …
>
> **2. Borrow Against the Position and Reinvest to Diversify**
>
> Another strategy is to borrow against your shares – up to 50% of their market value – and invest the loan proceeds in other securities. Borrowing generates liquidity without triggering a tax liability and may not trigger a reporting obligation to your company or the Securities and Exchange Commission. The key to success with this strategy is for the investments affected with borrowed funds to generate a higher return than the loan's interest rate and result in a more diversified portfolio. The current low cost of borrowing may make this strategy attractive. Of course, borrowing against or margining your portfolio securities entails special risks, including the potential that the securities may be liquidated in the event of market declines.

Contrary to Merrill Lynch's claim, the strategy to borrow against the concentrated position and buy additional securities virtually always *increases* risk. In this note, we explain the error of this "leveraged diversification" strategy. We also report results of simulations that demonstrate the virtual impossibility of reducing the risk of a concentrated position by leveraged investments in other stocks. Finally, we demonstrate how risk analysis can be implemented using a stylized case study.

---

[1] 3998 Fair Ridge Drive, Suite 250, Fairfax, VA 22030. www.slcg.com Dr. McCann can be reached at 703-246-9381 and Dr. Luo can be reached at 703-246-9382. We received helpful comments on an earlier draft from Jim Abernethy, Richard Himelrick and Maya Weil.

[2] http://askmerrill.ml.com/res_article/0,,17840,00.html. A scanned image of the web page is attached hereto as an exhibit.

In what follows, we demonstrate four major points.

1. Following a strategy of borrowing against a concentrated position and buying additional individual securities increases the investor's risk unless the returns to the additional securities bought are significantly negatively correlated with the returns to the concentrated position. As a practical matter, the necessary condition for reducing risk will not be met if both the concentrated investment and the additional securities purchased are common stocks.
2. The more similar the additional securities purchased are to the concentrated position, the riskier the resulting leveraged portfolio. In practice, the securities purchased to "diversify" were quite similar to the concentrated position. Often the strategy amounted to little more than making additional investments in the concentrated position on margin.
3. The risk of a leveraged portfolio increases with the amount invested in the additional securities. That is, the more an investor followed this bad advice, the worse the resulting portfolio.
4. In rare cases where the "leveraged diversification" strategy reduces risk, the leveraged portfolio's expected return is much less than the concentrated position's expected return. Thus the recommended strategy either increases risk or dramatically lowers the expected return of the concentrated portfolio - or it does both.

The Fallacy of Diversification Using Leveraged Investments

Superficially it appears that the advice to borrow against the concentrated position and buy additional securities was sound. Combining individual securities into portfolios reduces risk below the average risk of the individual securities while yielding the average return for the included securities.[3] Unfortunately for investors who followed this advice,

[3] The risk of a portfolio is lower than the average risk of the individual securities in the portfolio because above average returns experienced on some stocks in a portfolio are averaged with below average returns experienced on other stocks in the portfolio. The risk in a concentrated position that can be

rather than reducing risk as advertised, the strategy significantly increased risk because it leveraged the investor's equity.

A numerical example may help. An investor has $1,000,000, invested entirely in a highly volatile stock. There is a 70% chance that the stock's returns over the next month will be between +20% and -20%. Now, this investor borrows $1,000,000 and invests in a portfolio of other individual stocks. After buying $1,000,000 of additional stocks the range over which the portfolio's monthly returns will vary has narrowed to between +15% and -15%. The risk appears to be reduced but it has actually gone up 50% because now the returns are being earned on a $2,000,000 portfolio in which the investor only has $1,000,000 in equity. Before buying the additional securities there was a 70% chance that the investor's portfolio would be worth between $800,000 and $1,200,000 at the end of the month. After buying the additional securities the range of likely outcomes widens to from $700,000 to $1,300,000.[4]

Figure 1 illustrates the general point. Diversified portfolios are portfolios with the lowest risk for each level of expected return and form the *efficient frontier*. Individual securities and other imperfectly diversified portfolios plot below - or to the right of - the efficient frontier. The market portfolio plots on the efficient frontier. The concentrated

---

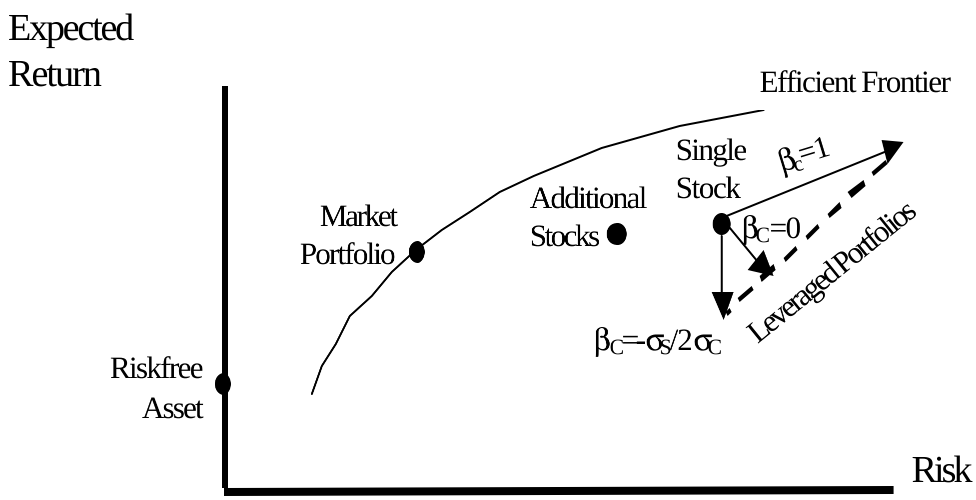
eliminated without a reduction in expected return is *diversifiable* or *uncompensated* risk. The investment risk in stocks and portfolios of stocks is measured using a statistical concept called standard deviation. The standard deviation of returns is expressed in percentage terms - the same as the returns themselves - and tells us how widely the observed returns fluctuate from day to day around their long run average. Modern portfolio theory is built on the concepts of portfolio expected return and standard deviation. For a detailed discussion *See* Fama (1974), Markowitz (1991), Bodie, Kane and Marcus (1993) or Sharpe, Alexander and Bailey (1995).

4 As can be readily seen from this example, the strategy reduces an investor's risk only if it reduces the volatility of the returns to the portfolio of securities by more than the amount of leverage taken on. If the investor's equity is leveraged 2 to 1 as in our example, the volatility of the enlarged portfolio's returns has to be half the volatility of the concentrated position's returns in order to reduce the investor's risk. Since the concentrated position still accounts for half the total securities portfolio value, such a reduction in the volatility is extremely unlikely.

It doesn't help much to have only partially leveraged up the investor's portfolio. If 50% of the value of the concentrated position was borrowed and used to purchase a complementary portfolio of securities the resulting portfolio's volatility would have to be one-third less than the volatility of the concentrated position and this is unlikely since the concentrated position is still two thirds of the enlarged securities portfolio. The fundamental error in the brokerage firms' strategy is a manifestation of the misuse of rates of return when dealing with portfolios of different size, which can be found in many settings. See Bodie, Kane and Marcus *Investments 4th Edition* p. 243-247 for a discussion of this error.

position an investor is holding plots far to the right.[5] The concentrated stock position typically has much more risk than the market portfolio and, in this example, it has a slightly higher expected return because it contains more market, or non-diversifiable, risk. The portfolio of additional stocks an investor buys with the margin debt when following the leveraged diversification strategy typically has more risk than the market portfolio but less risk than the single stock.

Figure 1
Risk and Expected Return of "Leveraged Diversification"



The investor in Figure 1 could sell some of the concentrated position and buy a combination of the portfolio of additional stocks, the market portfolio and the risk free asset. Diversifying so would result in a portfolio that is much less risky than the single stock with about the same expected return.

Figure 1 also illustrates the range of combinations of risk and return attainable by following the advice to borrow against a concentrated stock position and invest in a complementary portfolio of stocks. The dashed line represents the combinations of risk and return attainable by leveraging 100% of the value of the concentrated position and buying the portfolio of additional stocks. Exactly where along the dashed line the

[5] Throughout we will talk about this position as if it were a single stock but it could be any number of stocks so long as it is not well diversified.

investor ends up depends on the correlation between the concentrated position's returns and the returns to the additional stocks purchased. The detailed derivation of our results is contained in an appendix. We demonstrate next that diversifying a concentrated stock position with the purchase of additional stocks increases risk 99% of the time.

Simulations

As a theoretical matter, it was virtually certain that a strategy based on borrowing to buy additional securities would increase risk. We next report on extensive simulations we performed to demonstrate this conclusion empirically.

For each 100 stocks in the NASDAQ-100 at year-end from December 31, 1996 through December 31, 2002 we formed 10,000 value-weighted, 15-stock portfolios drawn randomly from the remaining 99 stocks resulting in 1 million portfolios for each year – 7 million portfolios in all. We then calculated the standard deviation of the daily returns to the single stock and the standard deviation of the returns to the leveraged portfolios formed by combining equal amounts of the concentrated position and of the matching 10,000 equally-weighted, 15-stock portfolios bought on margin. This scenario assumes that investors borrowed 100% of the value of the concentrated position or 50% of the value of the leveraged portfolios.

Only 1.4% of the 7 million fully leveraged portfolios had less risk than the concentrated positions. Thus, the strategy if implemented would have *increased* risk 98.6% of the time between 1997 and 2003.[6] On average, the fully leveraged portfolios were 44% more risky than the concentrated positions.[7]

We also performed simulations assuming that the investor borrowed 50% of the value of the concentrated position, or 33% of the value of the leveraged portfolios. Only 3.1% of the 7 million partially leveraged portfolios had less risk than the concentrated positions. On average, the partially leveraged portfolios were 26% more risky than the concentrated positions.

---

[6] Moreover the remarkable failure of this strategy was knowable. Looking back from December 31, 2001, only 147 of the 1,000,000 fully leveraged portfolios (i.e. 0.0147%) had less risk in 2001 than the concentrated position they were intended to diversify.

[7] 91% of the fully leveraged portfolios were 50% more risky than the concentrated position.

Figure 2 illustrates the gross error of not simply selling the concentrated position and buying a diversified portfolio. Over the sever year period from 1997 through 2003, the average risk of the concentrated positions was 68%. Fully diversifying the concentrated positions would have lowered the investor's risk, on average by 70%, from 68% all the way down to 20%.[8] Even selling only half the concentrated position and buying a diversified portfolio of stocks would have reduced the risk of the concentrated positions by 45%, from 68% down to 37%.

Figure 2
Merrill Lynch's Diversification Strategy
Led Investors Away From a Diversified Portfolio

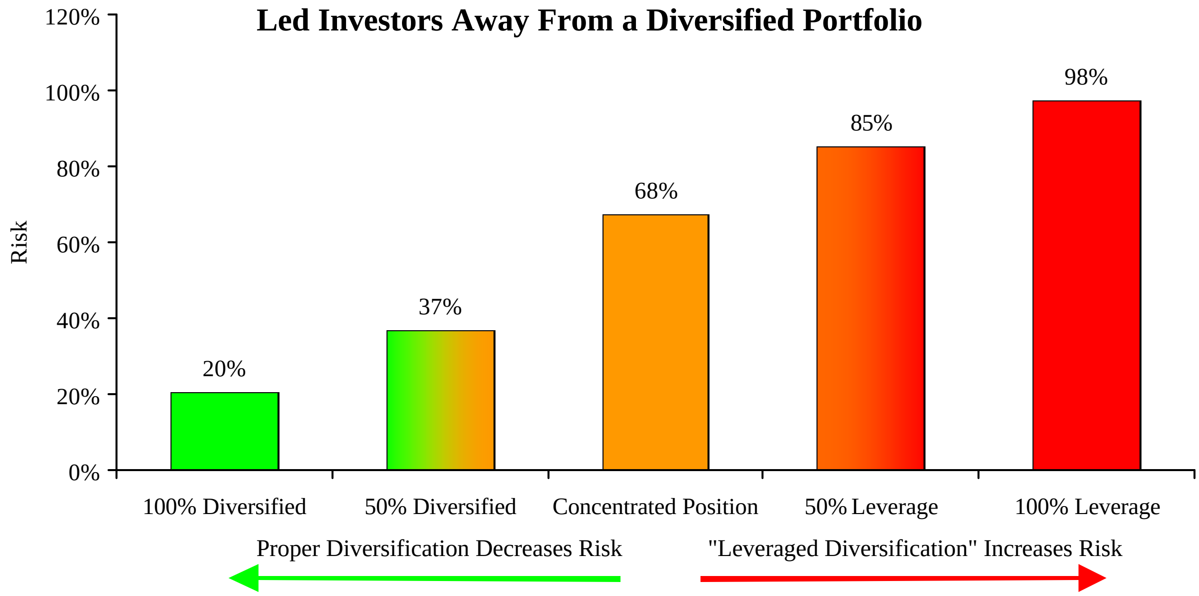


Figure 2 also illustrates the disastrous results of the leverage diversification strategy. Borrowing 50% of the value of the concentrated positions' value to buy additional stocks increased the investor's risk to 85% and borrowing 100% increased the risk to 98%. Our results confirm the theoretical investigation; following the "leveraged diversification" advice increased risk and the more investors followed the advice the worse the results. Following the leveraged diversification advice led to portfolios that were four to five times as risky, on average, as properly diversified portfolios.

[8] In our example, we use an index fund that tracks the S&P 500 to proxy for a fully diversified portfolio.

This dramatic result is not a function of the date we chose to start the analysis. Table 1 reports the results of our simulations for each year from 1997 to 2003. The leveraged portfolios were much more risky on average than the concentrated position in every year. In 1998 - the most favorable year for this strategy - it increased the investor's risk in 96.5% of the 1 million portfolios.[9]

Table 1
Summary of Risk Analysis

Year	Average Standard Deviation: 100% Diversified	50% Diversified	Concentrated Position	50% Leverage	100% Leverage	Fully Leveraged Portfolios More Risky Than Concentrated Position
1997	18.2%	29.6%	54.5%	60.2%	68.2%	96.6%
1998	20.3%	33.6%	59.4%	65.2%	73.9%	96.5%
1999	18.0%	34.2%	58.9%	63.8%	72.6%	97.3%
2000	22.2%	50.5%	91.2%	118.0%	137.7%	99.7%
2001	21.2%	44.0%	91.4%	133.0%	159.7%	100.0%
2002	26.0%	39.7%	78.4%	110.5%	116.9%	100.0%
2003	16.7%	27.2%	40.9%	47.1%	54.6%	100.0%
Average 1997 - 2003	**20.4%**	**37.0%**	**67.8%**	**85.4%**	**97.7%**	**98.6%**

The results in Table 1 reflect an interesting phenomenon. In years the stock market rose - like 1997, 1998 and 1999 and 2003 - the initial leverage declined over time and lessened but did not eliminate the perverse impact of this strategy on risk. In down markets like 2000, 2001 and 2002 the initial leverage grew over time exacerbating the perverse impact of this strategy on risk.[10]

[9] Our results are not the result of buying 15 individual stocks. We simulated the strategy 1 million times in 2002 and in 2003 using 10, 25 and 50 stocks for the complementary portfolio. The risk of the leveraged portfolio declines slightly the larger the number of stocks used in the complementary portfolio but the leveraged portfolio is even less likely to reduce risk as the number of selected stocks increases.

[10] If the securities portfolio's returns equal the rate of interest charged on the margin debt, the portfolio's leverage would remain constant. When we assume the amount of leverage remains constant rather than allowing it to vary with the returns to the portfolio we find the leveraged portfolio is more risky than the concentrated position 96.7% of the time.

Margin Calls

Investors following the leveraged diversification advice were often faced with margin calls as the value of their concentrated position and the value of the additional purchased stocks fell simultaneously. Our results confirm that these margin calls were the predictable result of the increased risk created by the strategy. During 2000, the investor's equity fell to 25% in 56% of our simulated fully leveraged portfolios. In 94% of the instances when the investor would have received a margin call, the value of the investor's concentrated position exceeded the net equity in the leveraged investment. That is, when a margin call would have been issued the investor would have almost always been better off not having leveraged up the concentrated position.

A Case Study: *Home Depot, Managed Accounts and Leverage*

We have demonstrated theoretically and empirically that a strategy of leveraging a concentrated stock position to purchase additional stocks virtually always dramatically increases risk. We now want to demonstrate how this analysis can be practically applied in a stylized case.[11]

On November 30, 2000 an investor owned 34,510 shares of Home Depot stock worth approximately $1,352,360, accounting for 99.3% of the investor's securities portfolio. By December 31, 2000 the Home Depot position was worth $1,576,676. After discussions with his broker, the investor borrowed $1,576,676 against the Home Depot stock and invested the proceeds in five accounts managed by professional money managers.

Table 2 shows the impact of this leveraged diversification strategy over time. The trailing 12-month annualized volatility of Home Depot stock was approximately 62% at the end of each month from November 2000 through June 2001. Purchasing an equal amount of other securities on December 31, 2000 reduced the volatility of the now

[11] Our discussion throughout the paper, and our empirical research, assumed the margin loan taken out against the concentrated position was used to purchase additional stocks. Our theoretical results apply equally well to situations where alternative investments, real estate for example, are purchased with the proceeds of the margin loan. Our simulations and case study presentation could be performed with such alternative investments.

enlarged portfolio to 38% but because of the leverage entailed by the strategy the investor's risk actually increased from 62% to 80%. A proper diversification strategy would have easily reduced the investor's risk all the way down to 22%. The leveraged diversification strategy thus left the investor with an account that was 4 times as risky as a readily achievable, well-diversified portfolio.

Table 2
Stylized Case Study Risk Analysis

	Portfolio Securities	Home Depot	Account Equity	Vanguard S&P 500 Index	Vanguard Total Stock Market Index Fund	NASDAQ Composite Index
11/30/2000	60%	60%	**60%**	**22%**	23%	46%
12/31/2000	38%	62%	**75%**	**23%**	24%	50%
1/31/2001	38%	63%	**74%**	**23%**	24%	52%
2/28/2001	38%	64%	**85%**	**22%**	24%	52%
3/31/2001	38%	63%	**88%**	**23%**	25%	53%
4/30/2001	38%	62%	**80%**	**22%**	24%	52%
5/31/2001	38%	62%	**78%**	**22%**	23%	50%
6/30/2001	37%	60%	**81%**	**22%**	23%	50%
Average	**38%**	**62%**	**80%**	**22%**	**24%**	**51%**

Conclusion

We have demonstrated both theoretically and empirically that a strategy advocated by financial advisors to reduce risk *predictably* did exactly the *opposite.* There are larger lessons for evaluating investment advice beyond the predictable failure of this strategy. Unless advice is hedged with such cautionary language as to make it ineffective as marketing, it will contain statements that will not likely stand up to careful scrutiny.

Appendix: Portfolio Mathematics

Risk

The standard deviation of a portfolio's returns, s_p, is given by Equation [1], where s_i is the standard deviation of security (or portfolio) *i*'s returns and σ_{ij} is the covariance between the returns to security (or portfolio) *i* and security (or portfolio) *j*.

[1] $$s_p = \left[\sum_{i=1}^{3} \sum_{j=1}^{3} X_i X_j s_{ij} \right]^{1/2}$$

For a three-security portfolio Equation [1] can be written out as Equation [2]

[2] $$s_p = \left[\sum_{j=1}^{3} X_1 X_j s_{1j} + \sum_{j=1}^{3} X_2 X_j s_{2j} + \sum_{j=1}^{3} X_3 X_j s_{3j} \right]^{1/2}$$

$$s_p = \left[\begin{array}{l} X_1 X_1 s_1^2 + 2 * X_1 X_2 s_{12} + 2 * X_1 X_3 s_{13} + X_2 X_2 s_2^2 \\ + 2 * X_2 X_3 s_{23} + X_3 X_3 s_3^2 \end{array} \right]^{1/2}$$

In the present context Equation [2] can be significantly simplified. Security 1 is the concentrated stock position, Security 2 is the complementary portfolio of securities and Security 3 is the margin debt. $X_1 = 1$, $X_3 = - X_2$, and $\sigma_{13} = \sigma_{23} = \sigma_3^2 = 0$ since the complementary securities are purchased with the margin loan and the interest rate charged on the margin loan is effectively fixed. Equation [2] reduces to Equation [3].

[3] $$s_p = \left[s_S^2 + 2 * X_C s_{SC} + X_C^2 s_C^2 \right]^{1/2}$$

For ease of exposition we perform the risk analysis in terms of portfolio variance rather than standard deviation. Since standard deviation is the positive square root of the variance any change in the variables that increases the variance also increases the standard deviation. Also for ease of exposition, we use the correlation coefficient between the returns to various investments rather than their covariance.[12] The correlation coefficient, r_{ij}, varies from –1 to +1 and indicates whether higher or lower than average values of one random variable are likely to occur when higher or lower than average values of another random variable occur.

There are a few special cases of interest. Substituting "1" in for r_{SC} and for X_C in Equation [3] we get Equation [3i]. Equation [3i] says that if the returns to the concentrated position and the diversification portfolio are perfectly correlated the standard deviation of the leveraged portfolio equals the standard deviation of the concentrated position plus the standard deviation of the complementary portfolio.

[3i] $$s_p = \left[s_S^2 + 2s_S s_C + s_C^2\right]^{1/2} = s_S + s_C$$

Substituting "0" in for r_{SC} and "1" for X_C in Equation [3] we get Equation [3ii]. Equation [3ii] says that if the returns to the concentrated position and the returns to the complementary portfolio are uncorrelated, the leveraged portfolio's standard deviation of returns is greater than the standard deviation of the concentrated position but not as great as if the standard deviation of the concentrated position plus the standard deviation of the complementary portfolio.

[3ii] $$s_S < s_p = \left[s_S^2 + s_C^2\right]^{1/2} < s_S + s_C$$

Substituting "-1" in for r_{SC} and "1" for X_C in Equation [3] we get Equation [3iii]. Equation [3iii] says that if the returns to the concentrated position and the returns to the complementary portfolio are perfectly negatively correlated, the leveraged portfolio's standard deviation equals the standard deviation of the concentrated position minus the standard deviation of the complementary portfolio.

[3iii] $$s_p = \left[s_S^2 - 2s_S s_C + s_C^2\right]^{1/2} = s_S - s_C$$

---

[12] $r_{SC} = {s_{SC}}/{s_S s_C}$.

Finally, substituting "$-\frac{s_C}{2s_S}$" in for r_{SC} and "1" for X_C in Equation [3] we get Equation [3iv]. Equation [3iv] specifies the correlation coefficient required for the leveraged portfolio to have the same risk as the concentrated position.

[3iv] $$s_p = \left[s_S^2 - s_C^2 + s_C^2\right]^{1/2} = s_S \Leftrightarrow r_{SC} - \frac{s_C}{2s_S}$$

Equation [4] derives our first general result. A leveraged investment in the complementary portfolio is riskier than the concentrated position if the correlation between the single stock's returns and the complementary portfolio's returns is not significantly negative. The correlation has to be more negative the larger the investment in the complementary portfolio and the more volatile the returns to the complementary portfolio relative to the concentrated position.

[4] $$\begin{aligned} &s_p^2 = s_S^2 + 2 * X_C r_{SC} s_S s_C + X_C^2 s_C^2 \\ &\therefore s_p^2 \geq s_S^2 \Leftrightarrow r_{SC} \geq -\frac{X_C s_C}{2s_S} \end{aligned}$$

Equation [5] derives our second result. The leveraged portfolio is riskier the greater the correlation between the returns to the concentrated position and the returns to the complementary portfolio.

[5] $$\begin{aligned} &s_p^2 = s_S^2 + 2 * X_C r_{SC} s_S s_C + X_C^2 s_C^2 \\ &\frac{\Delta\left(s_p^2\right)}{\Delta r_{SC}} = 2X_C s_S s_C \geq 0 \end{aligned}$$

Equation [6] derives our third result. The leveraged portfolio's risk increases with the fraction of the portfolio invested in the complementary portfolio unless the correlation between the single stock's returns and the returns to the complementary portfolio is significantly negative. That is, investing more in the complementary portfolio further increases the resulting leveraged portfolio's risk.

[6]
$$\frac{\Delta\left(\sigma_p^2\right)}{\Delta X_C} = 2\rho_{SC}\sigma_S\sigma_C + 2X_C\sigma_C^2$$
$$\frac{\Delta\left(\sigma_p^2\right)}{\Delta X_C} \leq 0 \Leftrightarrow \rho_{SC} \leq \frac{-X_C\sigma_C}{\sigma_S}$$

To this point our portfolio mathematics have assumed nothing about the expected returns of the portfolios. We have demonstrated that the leveraged investment in additional securities will increase the investor's risk unless the correlation between the concentrated portfolio's returns and the additional purchased securities' returns is extraordinarily negative.

Risk and Expected Return

For completeness we now examine the impact of the strategy on expected returns. For simplicity, we will assume that expected returns are generated from a single-factor model like the Capital Asset Pricing Model although the conclusions would be qualitatively unchanged if we assumed a more general return generating process.

The expected return to a portfolio of securities is equal to the weighted-average of the expected returns to the individual securities, where as above X_i is the fraction of the market value of the portfolio invested in security *i*. *See* Equation [7]. Equation [7] applies equally well to combinations of portfolios of securities.

[7]
$$E[R_p] = \sum_{i=1}^{3} X_i E[R_i]$$

A portfolio that contains a concentrated stock position, a margin loan and a complementary portfolio of securities will have an expected return given by Equation [8].

[8]
$$E[R_P] = E[R_S] + X_C E[R_C] - X_C R_{MarginLoan}$$
$$E[R_P] = E[R_S] + X_C\left(E[R_C] - R_{MarginLoan}\right)$$

For instance, if the portfolio is invested in a stock with an expected return of 16%, a complementary portfolio with an expected return of 12% and a margin loan at 8% the leverage portfolio will have an expected return of 20%.

We assume that the expected return on a security is determined by the security's degree of market risk, measured by its β, the return on risk free assets and the return above the risk free rate of return expected for the market as a whole. *See* Equation [9].

[9] $$E[R_i] = R_{Riskfree} + \beta_i \left(E[R_M] - R_{Riskfree}\right)$$

Substituting Equation [9] into Equation [8] we get

[10] $$E[R_P] = E[R_S] + X_C \left[R_{Riskfree} + \beta_C \left(E[R_M] - R_{Riskfree}\right) - R_{M\arg in}\right]$$

Earlier we showed that if $\rho_{SC} = -\frac{\sigma_C}{2\sigma_S}$ the leveraged portfolio has the same risk as the concentrated position. According to the CAPM securities returns are correlated through their sensitivities with the market portfolio, as follows.

[11] $$\rho_{SC} = -\frac{\beta_S \beta_C \sigma_M^2}{\sigma_S \sigma_C}$$

Substituting the condition on the correlation coefficient for the risk level to be unchanged into Equation [11] and rearranging to solve for β_C^* we get

[12] $$\beta_C^* = -\frac{\sigma_C^2}{\beta_S \sigma_M^2}$$

Substituting Equation [12] into Equation [10] we get Equation [13].

[13] $$E[R_P] = E[R_S] + X_C \left[R_{Riskfree} - R_{M\arg in} - \frac{\sigma_C^2}{\beta_S \sigma_M^2}\left(E[R_M] - R_{Riskfree}\right)\right] < E[R_S]$$

The term inside the square brackets is negative since the interest rate charged on margin balances is greater than the return on the risk free asset. When the leveraged portfolio has the same risk as the concentrated position, the expected return on the leveraged portfolio is significantly less than the expected return of the concentrated position. Thus the recommended strategy either increases the risk or lowers the return of the concentrated portfolio, or it does both.