Volatility Estimates

One of the most important characteristics of any asset is its volatility, or standard deviation of return. This value has several applications. It is used in assessing the risk of an asset, in the valuation of options or derivatives on an asset or in evaluating the range of investment returns within a Monte Carlo simulation.

Recall that the standard deviation of a set of numbers which in our case here will be the daily returns of an asset is calculated as follows. First the average of the returns is determined. This is referred to as the mean, **x**. The standard deviation is the then calculated as the expected value of the squares of the differences of each observation from this mean. Typically there is an adjustment to this value based on the frequency of the data that expresses the volatility on an annualized basis. Mathematically, the calculation goes like this; first we calculate the variance V as the expected value of these squared daily differences

$$V \quad = \quad S\ (x_i - x)^2\ / \ n.$$

The volatility is then calculated as the square root of this value times an annualizing adjustment, which for daily observations is approximately 16;

$$s \quad = \quad 16\ v\ V$$

Most of the time it is helpful to calculate the volatility using a more complicated recursive algorithm that places progressively less reliance on older data than newer data.

For some applications there is an options market that can provide information on the volatility needed. Corresponding to every observed price on an option, there is a value that will allow that price to be identical with the value produced by an options pricing model, Black-Scholes as an example. This value is called the implied volatility of the option and usually resembles the historic types of estimates discussed above. When this is not the case, as in cases where there is no options market, of the time period of the application differs from any market that does exist, statistical methods are necessary. It is important though that the estimate used be consistent with the application. The time period used to estimate the volatility is ultimately very important in determining the value that will be used.

Consider the following example. Imagine that in June of 2005, one needed to have an estimate for the volatility of Microsoft (MSFT). The value used should depend on the type of application, and in particular the time until maturity of that application, whether we are evaluating a short term option on MSFT or performing a complicated Monte-Carlo simulation that will run for many years.

In Chart 1 below we illustrate the price performance of MSFT for the last 5 years, and below that in Chart 2 is the volatility estimate for the stock that uses the prior 3 months of data to form each estimate.

Chart 1

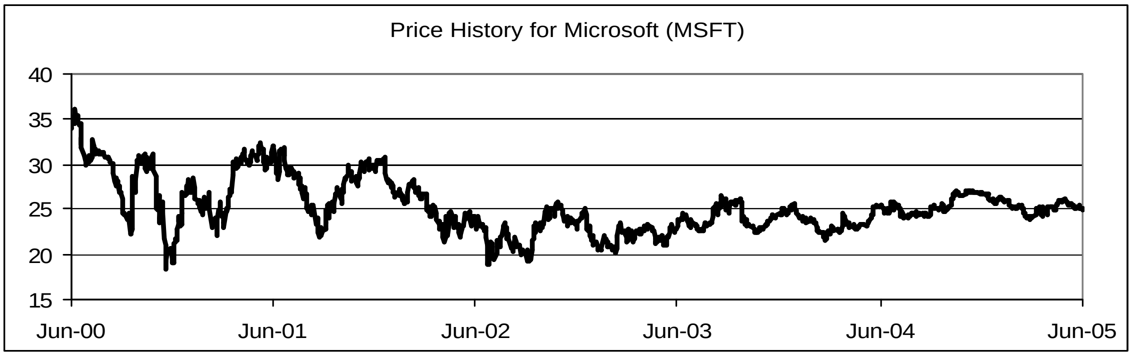
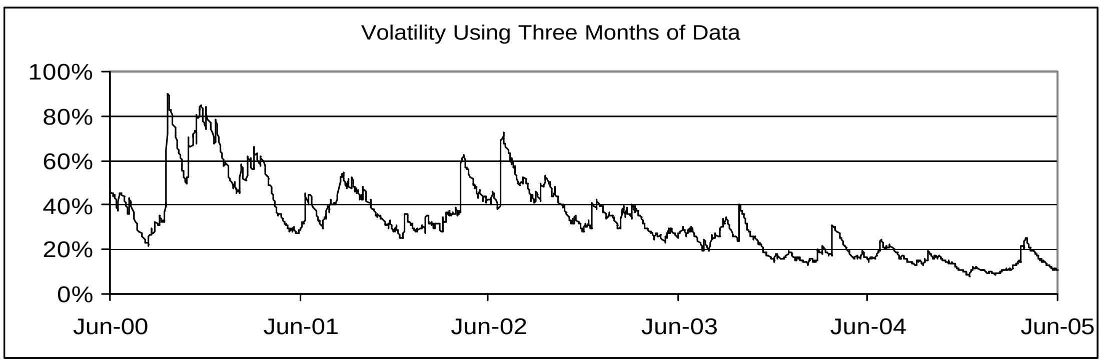
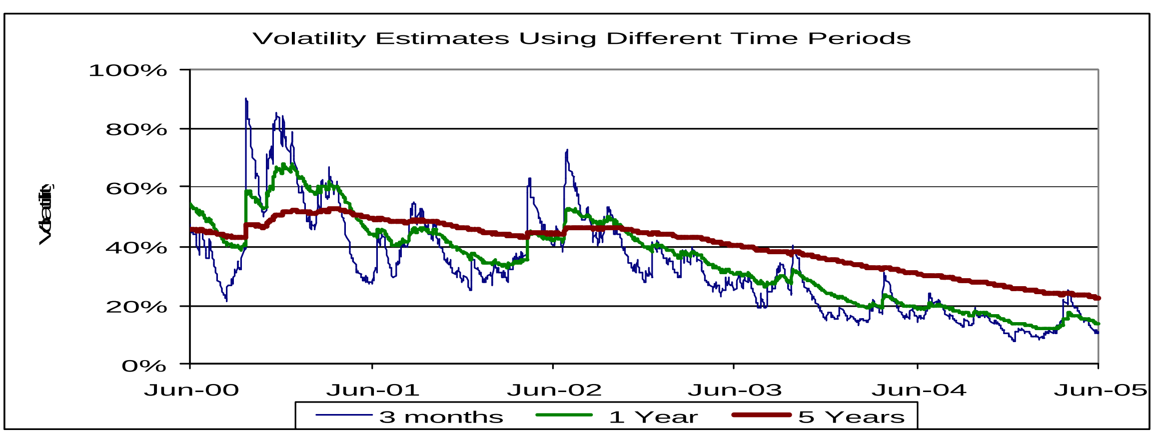


Chart 2

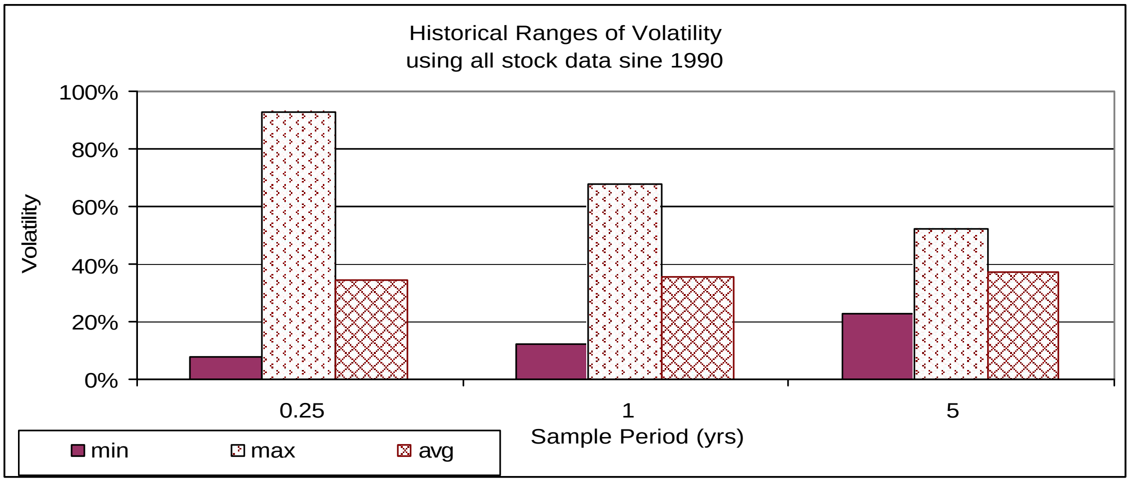


Even though the stock was in a price range of $20 to $35, this volatility estimate had a much wider range, from a low of 10% to a high of 90%. The implied volatility in short term options would have had a similar type of dispersion. The range though is very different when we use a longer history of data to evaluate the volatility. In Chart 3 we compare the volatility above with values calculated over 1 and 5 years of data.

Chart 3



As the estimation window gets larger, the range of observed values gets narrower as summarized in the next Chart:



The choice of the time period used to determine an estimate is very important. As an example, consider the employee stock option plan for Microsoft. According to the 2004 Annual Report, the average exercise price in the plan was $29, and the stock price was at $30. The volatility used to price these options was 30% which produces an average price of $10.21 per option. Their value of 30% corresponds to an estimate of long-term volatility. Chart 3 above shows what the volatility estimates would be using 5 years of stock return data. The average maturity of options in the Microsoft plan is about 7 years. At the volatility estimate of 10%, based only on recent data, the value of these options would only be $5.72.

In applications it is important to be aware that these two values can be quite different from each other and lead to very different results.

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