## Skewness and Kurtosis – Moving beyond Volatility

By Steve Pomerantz PhD

The notion of using standard deviation as a measure of risk has been received enormous acceptance throughout the investment community. Most of this success can be derived from centuries of academic analysis along with its simplicity in application. Based on the well known Normal or Bell-Curve distribution, it is both intuitive and easy to apply requiring either a basic textbook on statistics or any of several software packages easily available.

The basic idea, is that if a distribution of returns is Normal then the statistical measure called standard deviation, tells all that is needed for applications. For example, if the expected return is 0% and the standard deviation of daily returns is calculated to be 1% then we can make the following types of statements;

1.) 68.269% of the time the daily return will be in between -1.0% and 1.0%,
2.) there is a 0.135% chance that a daily return will be less than 3.0% or
3.) on average there will be 6 days a year when the return is less than -2.5%.

While the accuracy here may seem impressive, the reality is that data is rarely normal and in fact may deviate substantially. The next two most significant statistical measures that help to understand how the data differs from normal are Skewness and Kurtosis.

Skewness measures how symmetric the data is, in other words is there a tendency for the data to be positive or negative. Skewness measures the difference between the average and median of the data. The median measures the midpoint of the data, the value for which half the points are greater and half are smaller. For a symmetric distribution, like the normal, the median is the average and so the skewness is zero. If the skewness is negative, then there are more negative values and if the skewness is positive, then there are more positive values.

Kurtosis is a measure of extreme observations. How likely will the returns be extreme, either positive or negative. Though the sign of skewness is enough to tell us something about the data, kurtosis is often expressed relative to that of a normal distribution. Data that has more kurtosis than the normal is sometimes called fat-tailed, because its extremes extend beyond that of the normal. By definition, and according to the formulas used, the kurtosis of a normal distribution is 3.0. Fat-tailed distributions have values of Kurtosis that are greater than this.

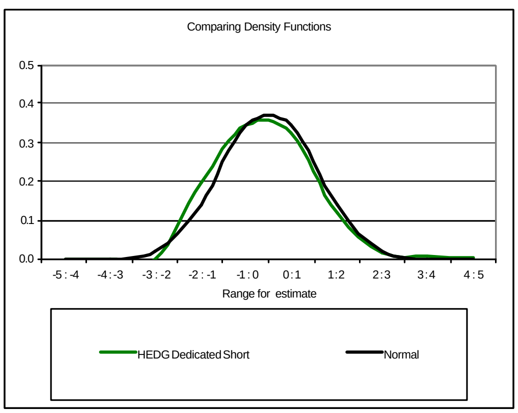
Using standard deviation alone to measure risk can be misleading if the returns being analyzed are too kurtotic as the following example will illustrate.

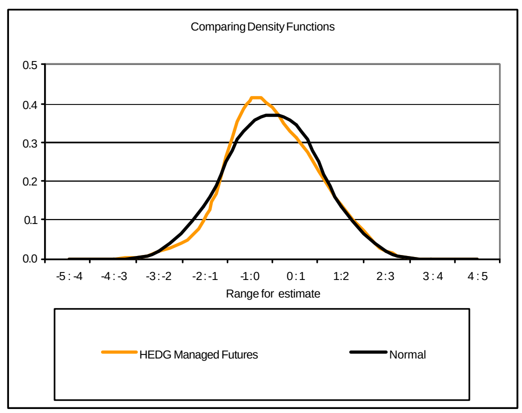
For this example we consider the monthly returns of several of the Tremont hedge fund indices. Tabled below is the data for the monthly return statistics from 1996 through 2005.

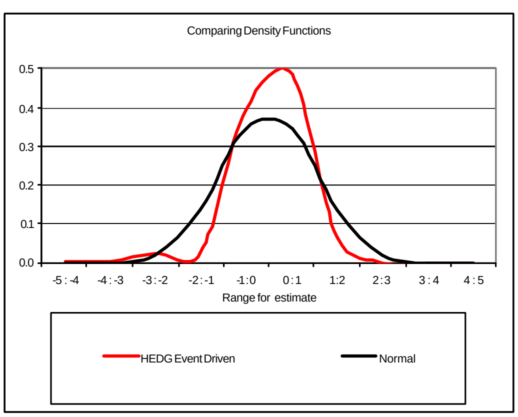
| | Mean | Standard deviation | Skewness | Kurtosis |
|---|---|---|---|---|
| Short Index | -.13 | 5.34 | -0.94 | 5.13 |
| Event Index | .88 | 1.76 | 3.78 | 27.95 |
| Futures Index | .62 | 3.54 | -0.12 | 3.07 |
| Hedge Fund Index | .83 | 2.17 | -0.22 | 6.65 |

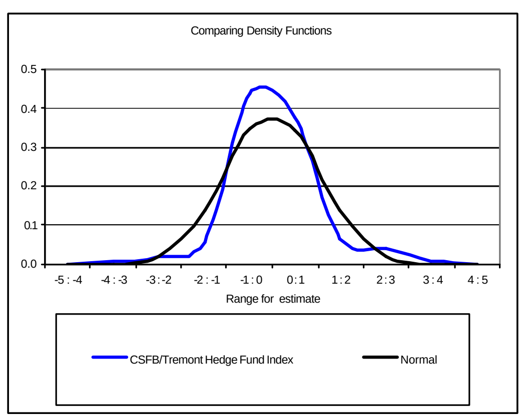
Note that only the Event Index has a positive skew value, the other values being negative. The Kurtosis of the Futures index is very close to that of the normal, with the rest being higher, and in the Event case, much higher.

One way to visualize these values is to normalize the returns and then graph the density functions. To normalize is to subtract from each series its average and then divide by the respective standard deviation. This converts each return into what is called a z-statistic. This is the value we are most familiar with in terms of normal distributions. These curves are displayed below. In each of the graphs below, the black line represents a standard normal density function, with a mean of 0.0 and a standard deviation of 1.0 while the colored line represents the standardized data for each of the hedge fund indices.



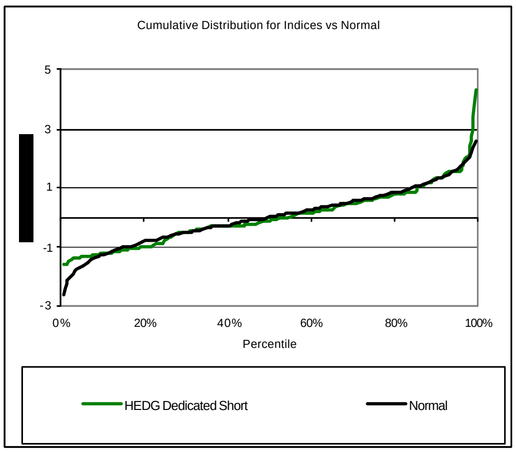



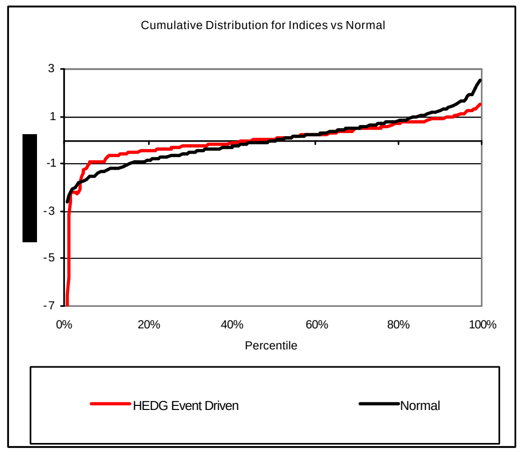



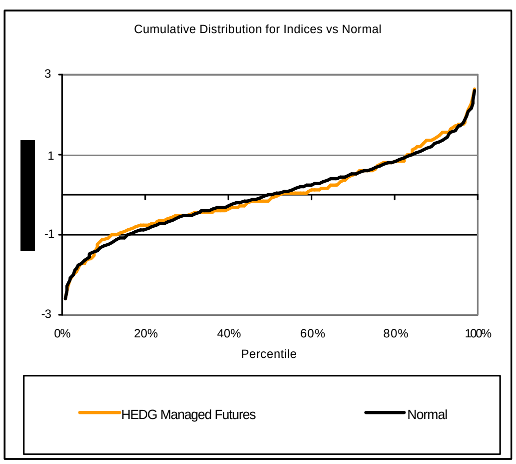
At the extremes, the Futures line is very close to the normal, representative of Kurtosis values very close to normal. The Event index has a very high kurtosis. This index peaks much higher than normal in the center but compensates for this by having more significant weight in the extremes, particularly to the left. The Hedge Fund index with the next highest kurtosis value has more weight on the right tail.

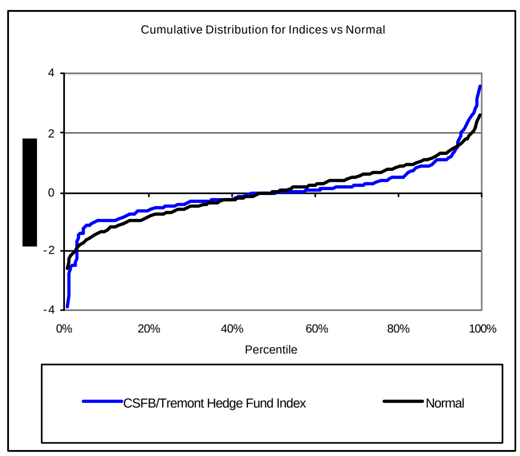
Comparing each line to the normal, we can see that the Event peak is to the right of the normal, but the other indices with negative skews have their peak to the left of normal.

An alternative way to represent this is with the cumulative density functions which look like the following




The left most points represent the lowest return in the time series'. Notice that the Event values on the left are significantly below the normal line, indicative of its very high kurtosis. The Future index with statistics close to normal lines up very closely to the black normal line.

What about risk? Kurtosis is very important when trying to measure risk or project worst-case scenarios. Most risk estimates, like VAR, are based on standard deviation to measure extreme possibilities.

As an example, consider the Event Index and try to estimate how the index performs under adverse conditions. Approximately 2.6 standard deviations cover the inner 99% of possible outcomes in a normal distribution. Based on these values for the event index, the 99.5% confidence estimate for a negative return would be given by subtracting 2.6 times the standard deviation from the mean, for a value of -3.69%. This means that there is only a 0.5% probability that a monthly return would be less than -3.69%. In reality though, given the actual data, the worst-case scenario under this definition (99.5% confidence) is -7.81%, almost twice as bad as would have been predicted from the normal model. Correcting the VAR for kurtosis, and skewness, would have produced a more accurate estimate of this worst-case value.

Whereas standard deviation is a quick, easy and time-tested measure of risk, for data that is highly kurtotic, it may only tell half of the story.

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