The Cost of Investing in Leveraged and Inverse ETFs

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Leveraged and Inverse Leveraged ETFs replicate the leveraged or the inverse of the daily returns of an index. Several papers have established that investors who hold these investments for periods longer than a day expose themselves to substantial risk as the holding period returns will deviate from the returns to a leveraged or inverse investment in the index. It is possible for an investor in a leveraged ETF to experience negative returns even when the underlying index has positive returns.

In this paper, we estimate distributions of holding periods for investors in leveraged and inverse ETFs. Using standard models, we show that a substantial percentage of investors may hold these short-term investments for periods longer than one or two days, even longer than a quarter.

We estimate the investment shortfall incurred by investors who hold leveraged and inverse compared to investing in a simple margin account to generate the same leveraged or short investment strategy. We find that investors in leveraged and inverse ETFs can lose 3% of their investment in less than 3 weeks, an annualized cost of 50%. We also discuss the viability of leveraged and inverse leveraged ETFs that rebalance less often than daily and calculate their costs to investors.

I. Introduction

Exchange-Traded Funds (ETFs) are similar to index mutual funds but are listed and traded on exchanges similar to unit investment trusts and closed end mutual funds. Unlike mutual funds, that trade only once a day at net asset value[2], ETFs trade at varying prices throughout the day just like stocks. State Street Global Advisors introduced the

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[2] For an in depth, discussion of the differences between index mutual funds and ETFs see for example Guedj and Huang (2010).

first ETF in the United States – the SPDR[3] that tracks the S&P 500 index – in 1993.[4] Since 1993, investments in ETFs have grown rapidly, from $66 billion in 2000 to $2 trillion in 2010 and their underlying portfolios have expanded beyond domestic stocks into bonds, foreign stocks, and commodities.[5] Investments in ETFs now account for about 40% of the total amount invested in index mutual funds in the US. Many stock exchanges around the world now also list ETFs. iShares, State Street and Vanguard are the three largest issuers of ETFs.

Investors can leverage purchases in or sell short ETFs in margin accounts subject to the same initial and maintenance margin rules that apply to purchases of most common stocks. Roughly speaking, the Federal Reserve Board's Regulation T prohibits the extension of credit to purchase common stock or the withdrawal of assets from a leveraged securities account that would reduce the investor's equity in the account below 50% of the value of the securities in the portfolio.[6] In addition, self-regulatory organizations ("SROs") require that member firms issue a margin call, i.e. demand addition unencumbered customer assets whenever the equity ratio in an account falls below 25% because of changes in the market value of the securities held in the account.

Leveraged and inverse ETFs combine traditional ETFs with internal borrowing or short selling to create simple leveraged or short investments. Until recently, investors in leveraged and short ETFs could make purchases that effectively leveraged or sold short an investment in securities without being bound by margin rules.[7] For example, a leveraged ETF portfolio manager might borrow 200% of the equity in her portfolio and invest 300% of the equity value in securities. The equity in the ETF portfolio in that situation is only 33% of the securities value. An investor that concentrated her account in

[3] SPDR - Standard & Poor's Depository Receipts, or "Spiders". SPDRs are the largest ETF by market capitalization.
[4] The first ETF was introduced on the Toronto Stock Exchange in 1990.
[5] See ICI Fact book (2010)
[6] http://ecfr.gpoaccess.gov/cgi/t/text/text-idx?c=ecfr&sid=40cc031a4a064ca8d3f500270b0d0fd7&rgn=div8&view=text&node=12:3.0.1.1.1.0.1.12&idno=12
[7] FINRA NTM 09-53 (2009) announced higher margin requirements for leveraged and inverse ETFs that take into account the underlying leveraged or short market exposures. In addition to avoiding margin requirements, leveraged and inverse ETFs allowed investors to gain leveraged or short exposure in retirement accounts.

a 3-1 leveraged ETF would effectively be using leverage that would not be allowed in a retail margin account.

The portfolio manager of an inverse ETF effectively replicates short sales that could also be done in a retail margin account. The inverse ETF portfolio manager effectively borrows and sells short investments in the reference index, earning interest on the portfolio's equity and the proceeds for the short sale and experiencing market returns opposite to the returns on the index. If the interest earned on the collateral assets net of fees is greater than the index returns, then the inverse ETF will have positive returns.

Leveraged and inverse open-end mutual funds similar to leveraged and inverse ETFs have been in existence for many years prior to 2006. For example, ProFunds' UltraBull (ULPIX) and UltraBear (URPIX) are open-end mutual funds launched in 1997 that leverage up or invert the daily returns to the S&P 500. Like leveraged and inverse ETFs, these mutual funds rebalance their portfolios to re-establish their target exposure ratios at the end of each day.

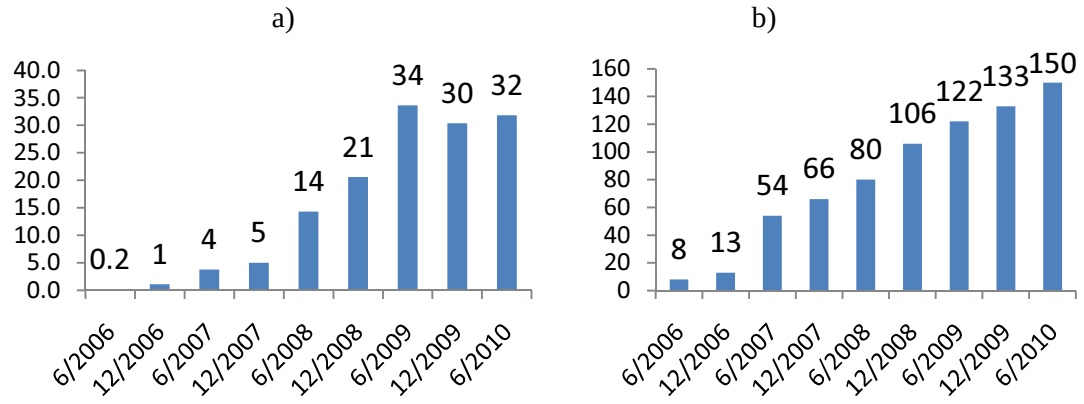
FINRA has issued a Notice to Members and additional guidance and the SEC has issued an Investor Alert about leveraged and inverse ETFs.[8] FINRA and the SEC have focused primarily on whether investors adequately understand that the returns to leveraged and inverse ETFs over holding periods longer than a few days are often significantly less than a multiple of the returns to the market index being referenced.

In this paper, we describe the problems associated with the daily rebalancing and the potential costs it may create for investors who hold these ETFs for periods longer than a few days. We use a methodology from the securities class action literature (see for example Barclay and Torchio (2001)) to infer the investors' holding periods from the observed trading volume. We apply this method to estimate the distribution of holding periods of investors in five different leveraged and inverse ETFs and use our results to calculate the shortfalls these investors have experienced compared to directly leveraging or selling short the underlying index with an ETF.

[8] FINRA Regulatory Notice 09-31 (2009), "Non-Traditional ETFs FAQ" at www.finra.org/Industry/Regulation/Guidance/P119781 and "Leveraged and Inverse ETFs: Specialized Products with Extra Risks for Buy-and-Hold Investors" at www.sec.gov/investor/pubs/leveragedetfs-alert.htm.

The ProFunds Group issued the first leveraged and inverse ETFs in the United States in June 2006.[9] There were 13 leveraged and inverse ETFs at the end of 2006, 66 by the end of 2007, and 153 by June 30, 2010. The total market value of leveraged and inverse ETFs has grown from $1 billion in 2006 to more than $30 billion by 2010. See Figure 1.

Figure 1: Number of leveraged and inverse ETFs and assets under management from inception in June 2006 to June 2010.



The growth in investments in leveraged and inverse ETFs since 2006 has occurred in part because of investments made by or on behalf of unsophisticated investors. These investors may not understand that a 200% or 300% leveraged ETF doubles or triples the underlying index returns only over very short holding periods and thus that these leveraged ETFs are likely to return substantially less than double or triple the underlying index returns over holding periods longer than a few days or weeks. In fact, counterintuitively, as a result of daily rebalancing of the leveraged and inverse ETF portfolios to re-establish the same leverage or short ratio at the end of each day, both 200% and 300% leveraged ETFs and inverse ETFs are quite likely to have negative returns across long holding periods whether the underlying market returns are positive or negative.

[9] "ProFunds Readies ETFs That Leverage Indexes," Investor's Business Daily, 26 May 2006.

Table **1** lists the number of leveraged and inverse ETFs and market value by issuer as of June 30, 2010. The three primary issuers – ProFunds Group ("ProFunds"), Direxion Funds ("Direxion") and Rydex Investments ("Rydex") - are mutual fund companies that had previously concentrated on active mutual fund traders and investment advisors. Together they account for 98% of the market capitalization of leveraged and inverse ETFs.

Table 1: Leveraged and Inverse ETFs by Issuer, June 30, 2010

	Leveraged ETFs			Inverse ETFs			Leveraged ETFs	
	Number	Assets ($millions)		Number	Assets ($millions)		Number	Assets ($millions)
ProShares	42	$7,020		60	$18,379		42	$7,020
Direxion	17	$3,280		17	$2,290		17	$3,280
Rydex	7	$134		7	$148		7	$134
Other				3	$600			
Total	66	$10,434		87	$21,416		66	$10,434

Wang (2009), Cheng, Minder, and Madhavan (2009), Wong and Hargadon (2009), and Little (2010) show how daily rebalancing back to a specified leverage or short ratio requires leveraged and inverse ETF portfolio managers to buy at the end of days when the underlying market is up and sell at the end of days when the market is down.[10] When daily market returns are volatile but the realized returns over longer holding periods are close to zero, this rebalancing has the effect of repeatedly buying high and selling low. The more volatile the daily returns the greater the losses suffered by leveraged and inverse ETFs in compared to the leveraged or inverse returns to the market.

The paper proceeds as follows. Section II explains the mechanics of the daily portfolio rebalancing. We highlight the cost inherent in daily rebalancing using as examples, Direxion's Leveraged and Inverse Financial Services ETFs. Section III calculates the investment shortfalls incurred by unsophisticated investors. Section IV describes the investors' investment horizon and the possibility of developing investments that would be more suitable for their holding periods. We conclude in Section V.

[10] See Zweig (2009) and Laise (2009) for discussions in the Wall Street Journal of the issue.

II. Rebalancing, compounding and holding period returns.

Leveraged and inverse ETFs internally rebalance their long and short positions at the end of each day so that the leverage or short ratio is the same at the beginning of each day as it was at the initial public offering. .

Table 2 presents a simple, five-day example of the impact of rebalancing and compounding on leveraged and inverse ETF returns. The daily returns accumulate over the five days to 0.01%.

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Table 2: Example of the impact of rebalancing and compounding on ETFs.

	Index Returns		Traditional ETFs and Cash or Margin Debt			Leveraged and Inverse ETFs	
	a)	b)	c)	d)	e)	f)	g)
Day	Daily Return	Cumulative Return	Unlevered ETF	$200 cash, short $100 ETF	$200 margin, $300 ETF	1X I-ETF	3X L-ETF
0			$100.00	$100.00	$100.00	$100.00	$100.00
1	23%	23.00%	$123.00	$77.00	$169.00	$77.00	$169.00
2	-20%	-1.60%	$98.40	$101.60	$95.20	$92.40	$67.60
3	20%	18.08%	$118.08	$81.92	$154.24	$73.92	$108.16
4	-23%	-9.08%	$90.92	$109.08	$72.76	$90.92	$33.53
5	10%	0.01%	$100.01	$99.99	$100.03	$81.83	$43.59

Columns c), d) and e) reflect the returns that would be earned gross of fees and expenses in a traditional ETF, from selling short a traditional ETF and from leveraging up a traditional ETF 3-to 1. Column c) reports the value of $100 invested in an unlevered ETF at the beginning of the week is $100.01 at the end of the week. Column d) shows that the same $100 used to collateralize a short sale of $100 would end the week worth $99.99 ignoring any interest that might be earned on the proceeds of the short sale. Column e) shows the result of leveraging up a traditional ETF 3-to-1 over the same 5-day period. We have constructed this example so that the 5-day holding period returns to the unleveraged ETF and to the leveraged and short investments in the ETF are all 0%.

Columns f) and g) report the results of selling short the underlying market by investing in an inverse ETF and the result of leveraging up the underlying market by investing in a 3X leveraged ETF. Column f) shows the result of investing $100 in an inverse ETF over the same week. Initially the inverse ETF portfolio holds a $100 short position in the index, $200 in collateral and $100 net equity. The resulting market

exposure is -$100. On the first day, the market return is 23%, the inverse ETF's return is -23%, and the first day, the 1X inverse ETF investment is worth $77 since its underlying short position in the index is now $123 liability against its $200 in cash. The inverse ETF's market exposure is now -160% (i.e. -$123/$77 = 160%).

If the inverse ETF portfolio manager does not adjust the portfolio, the returns on the inverse ETF for the second day will equal -1.6 times the returns on the index. Instead, to re-establish a -100% market exposure to start the second day, the portfolio manager uses $46 of the $200 in cash to reduce the -$123 end-of-day short market exposure down to -$77. The -20% index return on the second day reduces this short market exposure from -$77 to -$61.60 and the inverse ETF value increases to $92.40 at the end of the second day. Of course, now at the end of the second day, the exposure is again no longer -100%. Because of the decrease in the value of the short position and the increase in the net asset value, the exposure ratio has fallen to 66.67%. To restore exposure to 100%, the inverse ETF manager must now increase back the leverage to 100% by increasing the short exposure by $30.80.

This simple example highlights the "constant leverage trap". Over a 5-day period, the ETF returns, as well as the leverage, and a short investment returns in the ETF was 0%. However, an investment in a 1X inverse ETF lost 18.2% and an investment in a 3x leveraged ETF lost 56.4%, these substantial losses are due to the daily compounding these investments do that can end up being very costly for long-term investors.

Wang (2009) shows that the return on a leveraged ETF is:

$$(1 + R_T^{L-ETF}) = \left(1 + R_T^{index}\right)^x \cdot e^{\frac{(x-x^2)\sigma^2 T}{2}}$$

Where x is the leverage ratio, σ is the volatility of the index, and T is the time period the investment is held. For all leveraged ETFs in the market, the scalar term $e^{(x-x^2)\sigma^2 T/2}$ is positive and less than one and goes to 0 the longer the holding periods. Hence, the return of the leveraged ETF is a function of the return of the underlying index to the power of the leverage, time a multiplier that is less than 1 and goes to 0 over time. Thus, if the volatility is high, or the holding period is long, the constant will be small,

potentially making the return on the leveraged ETF smaller than that of its underlying index. This illustrates why it is possible for an investor in a leveraged ETF to experience negative returns even when the underlying index experienced positive returns.

The daily rebalancing of leveraged and inverse ETFs creates a situation that for periods longer than a day or two the return of a leveraged or inverse ETF will deviate from the margin account benchmark. The magnitude of the deviation will depend on the index characteristics for the holding period, mainly its volatility and its path. The higher the leverage and the longer the time period the more likely it is that this deviation will be more substantial. In general, as long as the underlying index has no clear trend, the higher the volatility, the higher the leverage, and the longer the time period, the more the investor will lose compared to investing in a leveraged or short position using a margin account.

The magnitude of the investment shortfall in leveraged or inverse leveraged ETFs and a margin account over long periods can be substantial. However, the early ETF prospectuses did not fully explain the investment shortfall always warn that investors should exercise extra caution when investing in these funds. For example, one such prospectus stated, *"The Fund's current benchmark is 200% of the performance of the S&P 500 Index (the "Index" or "Underlying Index"). If the Fund meets its objectives, the value of the Fund's shares will tend to increase on a daily basis by 200% of the value of any increase in the Underlying Index"*[11]*;* or another one stated that "*The correlations sought by the Bull Funds and the Bear Funds are generally a multiple of the returns of the target index or benchmark.*"[12]

Direxion issued (3x) and (-3x) ETFs in November 2008. Their prospectus on September 29, 2008 stated,[13] "*The Funds described in this Prospectus seek to provide daily investment results, before fees and expenses that correspond to the performance of a particular index or benchmark. The Funds with the word "Bull" in their name (collectively, the "Bull Funds") attempt to provide investment results that correlate positively to the return of an index or benchmark, meaning the Bull Funds attempt to*

[11] www.sec.gov/Archives/edgar/data/1208211/000093506906003020/g36000_etf485a.txt
[12] www.sec.gov/Archives/edgar/data/1424958/000089843208000978/direxion.htm
[13] www.sec.gov/Archives/edgar/data/1424958/000089843208000978/direxion.htm

move in the same direction as the target index or benchmark. The Funds with the word "Bear" in their name (collectively, the "Bear Funds") attempt to provide investment results that correlate negatively to the return of an index or benchmark, meaning that the Bear Funds attempt to move in the opposite or inverse direction of the target index or benchmark. The correlations sought by the Bull Funds and the Bear Funds are generally a multiple of the returns of the target index or benchmark."

These statements illustrate how confusing descriptions of leveraged and inverse ETFs may be, and that they do not always clearly explain that ETFs are not suitable for investors with investment horizons longer than one day. It was not until 2009, after many leveraged ETFs suffered significant losses while the reference ETFs had gains, that Rydex and Proshares ETFs improved their disclosures[14]. For example, on December 16, 2009[15], Rydex's new prospectus emphasized the *daily leveraged* investment goals and stated it is not suitable for "*investors who do not intend to actively monitor and manage their portfolios*." Proshares, in their prospectus on June 23, 2009[16], addressed the investor suitability issue in a separate paragraph on two new products and then on all of the leveraged and inverse leveraged ETFs in their July 31, 2009 prospectus[17].

Direxion's Leveraged and Inverse ETFs Financial Services

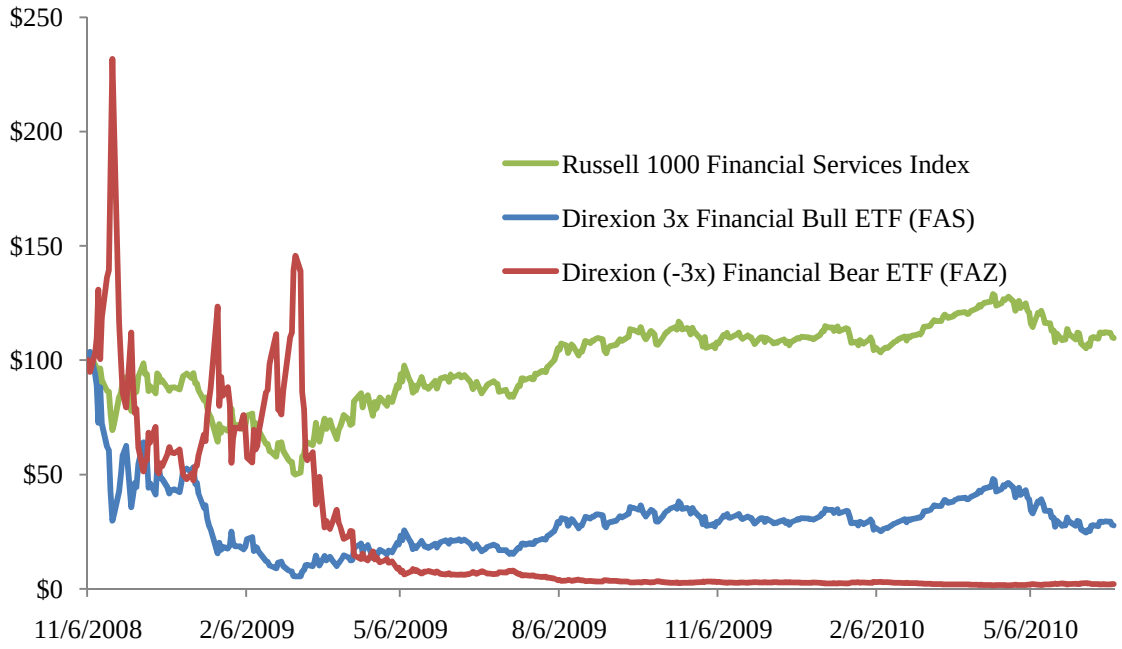
We illustrate the observed pattern of returns to investing in leveraged and inverse ETFs using a pair of ETFs issued by Direxion, both based on the Russell 1000 Financial Services Index (RGUSFL).[18] Direxion Financial Bull 3X ETF (FAS) and Direxion Financial Bear 3X ETF (FAZ) were both first issued on November 6, 2008. FAS leverages up an investment in the financial services sector 3-to-1 each day, for one day. FAZ sells 300% of the fund's net assets short in the financial services sector each day, for

[14] Direxion emphasized on investor suitability in their prospectus filed on December 17, 2008.
[15] http://sec.gov/Archives/edgar/data/1208211/000089180409005431/sb47870-485b.txt
[16] Proshares prospectus on June 23, 2009: http://sec.gov/Archives/edgar/data/1174610/000119312509135520/d485bpos.htm
[17] http://sec.gov/Archives/edgar/data/1174610/000119312509160939/d485apos.htm
[18] As of May 28, 2010, RGUSFL 10 largest constituents were JPMorgan Chase, Bank of America, Wells Fargo, Citigroup, Goldman Sachs, US Bancorp, American Express, Morgan Stanley and Visa. www.russell.com/indexes/PDF/fact_sheets/US/1000Financialservices.pdf

one day. With some minor variation, the daily return to FAS will be equal in size but opposite in sign, to the daily return to FAZ.

Figure 2 plots the value of an investment of $100 in FAS, FAZ and the Russell 1000 Financial Services Index (RGUSFL). Investors in the two Direxion ETFs suffered significant losses from inception to any time after the first four months, even though the financial services index is up during this period.

Figure 2: Direxion's FAS, FAZ, and the Russell 1000 Financial Services Index from November 6, 2008 to June 23, 2010.



Investors who thought that FAS or FAZ were effective ways to make any more than transitory bets on the direction of the financial services industry might be shocked by the returns illustrated in Figure 2. The Russell 1000 Financial Services Index gained 10% over this period, yet FAS, the (3X) leveraged ETF, rather than returning 30% lost 72.4% and, the (-3X) inverse leveraged ETF, FAZ, rather than losing 30%, lost 97.9%. The counterintuitive pattern illustrated in Figure 2 is common for leveraged and inverse ETFs and results from the daily rebalancing of the funds portfolio.

III. Potential Shortfalls Incurred by Long-Term Investors

Unsophisticated investors who don't understand that leveraged ETFs are a poor way of leveraging or selling short an index for a period longer than a day or two may have experienced substantial investment shortfalls compared to having invested in a margin account. The extent of the shortfall depends on the holding period of the investment and the returns and volatility of the underlying ETF. In order to precisely calculate the investment shortfalls caused by the mismatch between investors' investment horizon and the fund's daily horizon we need to observe the actual holding periods of the investors. As these holding periods are not publicly available, we use the trading models commonly used in establishing damages in securities class action litigation to estimate the investors' holding periods. Barclay and Torchio (2001), McCann and Hsu (1999), and Beaver, Malernee and Keeley (1997) among others describe the methodology of using Trading Models and their advantages and shortcomings.

The simplest model, the Proportional Trader Model ("PTM"), assumes that each share outstanding is equally likely to trade. Thus, shares which trade each day are drawn from those which have recently traded and those which have not recently traded in proportion to the relative size of these two groups. For example, assume there are 1,000 shares outstanding and on day 1 we observe 200 shares traded. The PTM assumes that each investor sells proportionally 20% of their shares and are left with 80% of their previous day's holdings. On day 2, we observe 100 shares are traded and so we assume that investors have sold 10% of their remaining shares. The PTM repeats this process for the time period of interest and is thus able to estimate the distribution of holding periods time for each day's purchase and the price at which they are bought and sold. Murray and Belfi (2005) argue that the PTM method meets the legal criteria set by the Supreme Court for admission as a valid legal method for calculating damages.

The Multiple Trader Model ("MTM") assumes that there are at least two types of investors whithin each trader type with a different level of trading activity. Shares outstanding trade and daily trading volume are allocated among these types of traders and the PTM is applied to each type separately. The separate PTM results are then added together to arrive at total estimated damaged shares. Barclay and Torchio (2001) compare

different variations of the proportional trading model to demonstrate that results from the proportional trading model can be consistent with the results of multi-trader models when certain assumptions and parameters are used. The MTM model appears to be appropriate for our research since a part of the ETF trades are done by market makers and arbitrageurrs and only a part is done be by individual investors. See Appendix I for a detailed description of the procedures we follow.

We illustrate our methodology for estimating investment shortfalls with five leveraged and inverse ETFs listed in Table 3. We use a cross section of ETFs from three different issuers, with a variety of positive and negative leverages, spanning different indexes from equity indexes, broad indexes, and bond indexes.

Table 3: List of five leveraged ETFs for which we calculate shortfalls

Ticker	Full Name	Issuer	Leverage	Index
DPK	Direxion Developed Markets Bear 3X - Triple-Leveraged ETF	Direxion	-3	MSCI EAFE Index
TYO	Direxion 10-Year Treasury Bear 3X - Triple-Leveraged ETF	Direxion	-3	NYSE Current 10 Yr US Treasury Index
RHO	Rydex Inverse 2X S&P Select Sector Health Care ETF	Rydex	-2	AMEX Health Care Select Index
SBB	ProShares Short Small Cap 600 Fund	ProShares	-1	CBOE S&P Small cap 600 Index
UVG	ProShares Ultra Russell 1000 Value Fund	ProShares	2	Russell 1000 Value Index

Table 4 describes the basic results of our analysis. We show the average turnover ratio for each ETF since inception and the estimated distribution of investors' holding periods. The average daily turnover ratio is a basic indicator of the average holding periods of an ETF. However, the MTM method allows us to estimate more than the average holding period, it estimates the distribution of holding periods. As

Table 4 illustrates, even funds that have a high daily turnover ratio will have some investors holding the fund for longer periods than a few days. We describe the distribution by showing the percentage of investors who hold a fund longer than a week,

a month, and a quarter. All five ETFs in our sample have a substantial percentage of investors that have holding periods longer than a month, ranging from 6% to almost 24% of the investors. More than 8% of the investors in UVG hold the ETF even longer than a quarter.

Table 4: Calculated holding periods of five leveraged ETFs.

ETF	Average Daily Turnover Ratio	Leverage Ratio	Average Holding Period (days)	Purchases Held for More Than 1 Week	Purchases Held for More Than 1 Month	Purchases Held for More Than 1 Quarter
DPK	18.1%	-3	5.3	16.42%	6.30%	1.22%
TYO	5.5%	-3	12.8	48.02%	16.39%	3.89%
SBB	4.6%	-1	21.4	55.49%	21.62%	8.50%
RHO	2.9%	-2	18.4	61.28%	27.62%	6.58%
UVG	3.7%	2	22.7	54.31%	23.91%	8.90%

In general the daily turnover ratio is invertly correlated with the average holding periods. Hence SBB, RHO and UVG that have lowew turnover, have higher average holding periods. This translates, in general, also to the distribution of holding periods too. However, the results of the distribution of the holding periods highlights the strength of the MTM method and the importance and contribution of using it to infer the investors' holding periods. RHO has the lowest average daily turnover, but once we take into account the actual daily volume we infer that the average daily holding period is long (18.4 trading days) but not necesserally as long as UVG for example. Nonetheless more than 60% of the investors hold RHO longer than one week, while only more than 54% hold UVG longer than a week. The MTM method gives us an accurate tool to infer the actual distribution, with is very important when trying to analyze the behavior of investors that are using an investment that is designed to be held only for the short-term.

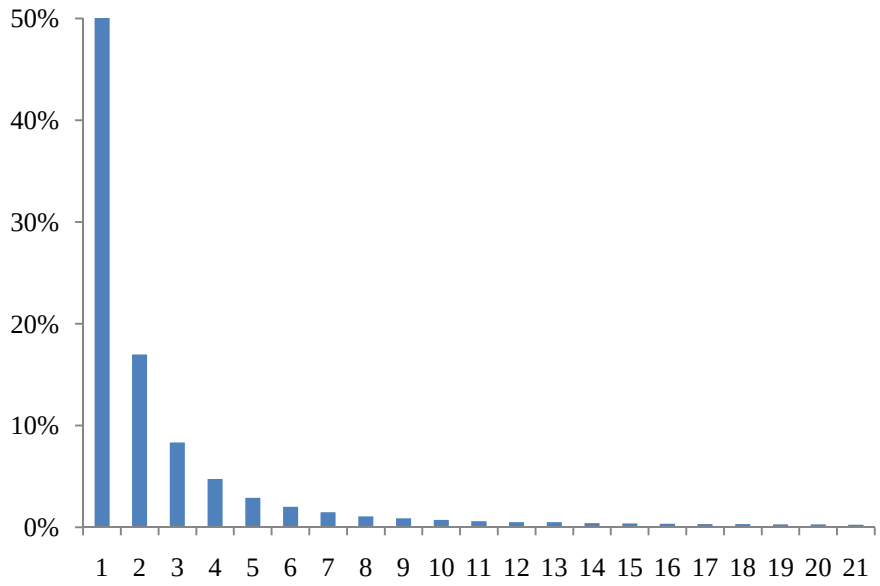
The percentage of investors that hold these investments longer than a month is striking. More than 27% of the investors hold RHO longer than a month as do more than 23% of the investors in UVG. More than 8% of the investors hold SBB and UVG longer than a quarter, a very long time for an investment that should be held a day or two.

As shown in Table 4, DPK has a high daily turnover of 18.1%. We will use this ETF to illustrate our findings in this paper, as it has the highest daily turnover ration, and

the lowest average holding period. Investors in ETFs with lower turnover ratios are likely to hold their ETFs for longer time periods and potentially could have higher shortfalls. A high turnover and a low average holding period does not necesseraly translate to a narrow distribution of inevstors' holding periods. Figure 2 contains a histogram of the distribution of the investors' holding periods of Direxion Daily Developed Markets Bear 3x (DPK) that offers 3x the daily return the MSCI EAFE Index[19]. For DPK, more than 16% of the investor held the fund longer than a week, more than 6% of all investor held the fund longer than a month, and more than 1% of the investor held it longer than a quarter.

DPF's prospectus states "*the Funds are designed as short-term trading vehicles for investors who intend to actively monitor and manage their portfolios.*[20]" Despite this statement, it appears that more than 6% of investors held this ETF longer than a month. This indicates that many investors do not understand the inherent cost associated with holding a short-term investment for a long-term.

Figure 2: Distribution of holding periods in trading days of DPK.



[19] The MSCI EAFE Index is a stock market index that is designed to measure the equity market performance of developed markets (Europe, Australasia, Far East), excluding the US & Canada. See, http://www.mscibarra.com/products/indices/international_equity_indices/definitions.html#EAFE

[20] www.sec.gov/Archives/edgar/data/1424958/000089843210000945/a485bpos.htm, page 30.

We use the MTM methodology from the securities class action literature to estimate the investment shortfall or the difference between holding the ETF and holding the index (in the form of the index ETF) in a margin account. In Figure 3, we present the difference between the two strategies' holding period returns as a function of the number of days the investor held DPK. For all holding periods, DPK on average had lower returns than its benchmark. Moreover, the longer the holding period, the greater the investment shortfall from the benchmark. On average, an investor that held DFK for 15 trading days (3 weeks) lost 3% of her investment compared to the benchmark. In other words, had the investor created the leverage themselves in a margin account they would have earned 3% more over a 3 week time period, the equivalent of more than 50% on an annualized basis. For high volatility indexes, the longer an investor holds a leveraged ETF the greater the investment shortfall compared to a non-rebalanced margin portfolio.

Figure 3: Holding Period Return Differences by Days (Margin Account – DPK)

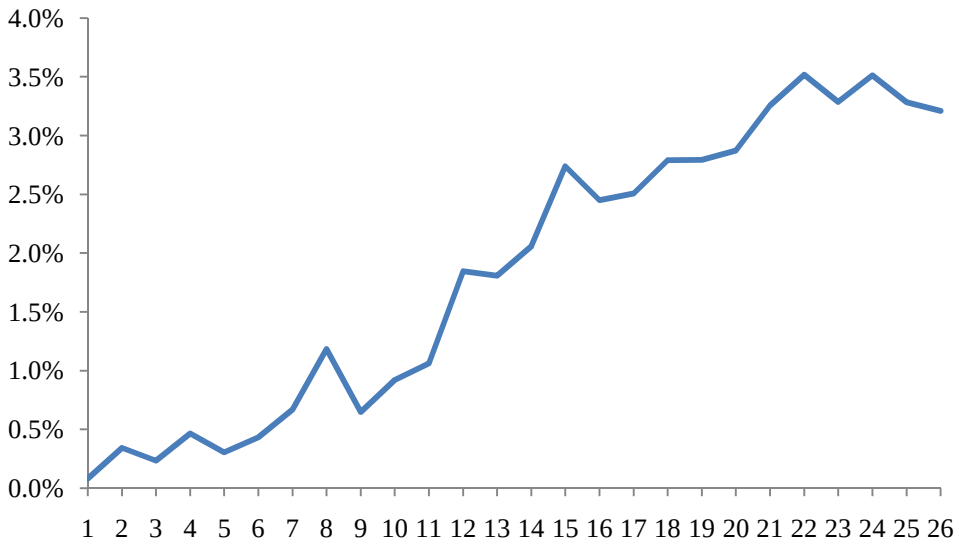
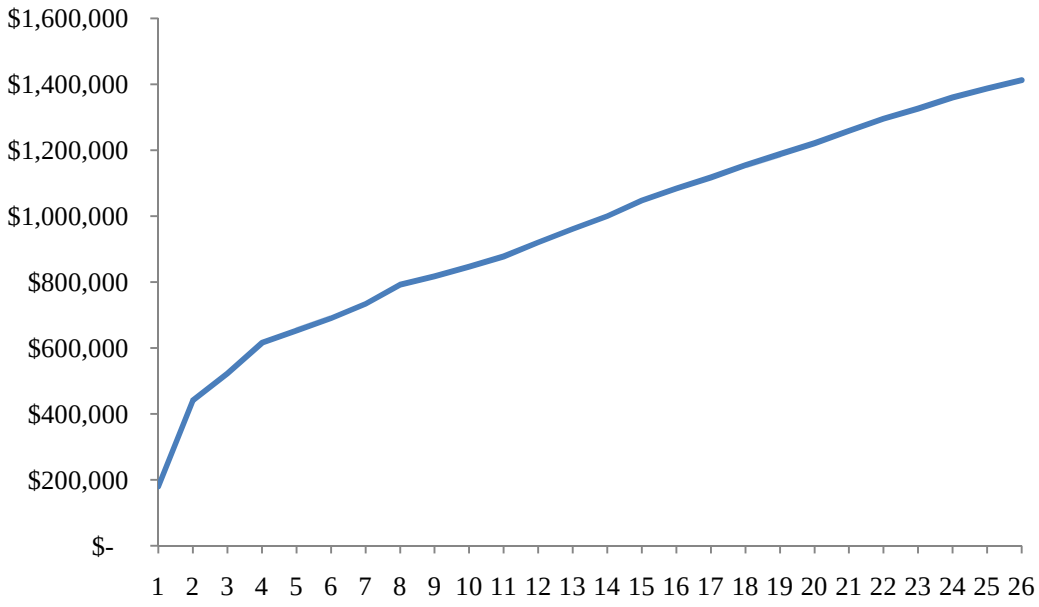


Figure 4 shows the total amount in dollar terms of the cummulative shortfall incurred by investors in DPK as a function of the number of days they held the fund. We calculate that investors in DPK lost at least $1.8 million since the insception of the fund in December 2008 compared to an investment in a the benchmark portfolio. This ammount is substantial as at its inception DPK had a market capitalization of only $6 million, and the avearge market capitalization of only about $8.5 million since inception.

Figures 3 and 4 illustrate two important facts about investors holding these ETFs for the long-term. First, there can be substantial shortfall for investor even when holding the investment for three or four days. On average, the investors in DPK lost 0.5% of their investment holding it for 4 days, more than 30% loss annualized. Second, the investors holding DPK for up to four days amount to a total shortfall of about $600,000, a substantial percentage of the total shortfall of $1.4 million dollars.

Figure 4: Holding Period Cumulative Total Shortfall (Margin Account – DPK)



In Table 5, we show the calculated shortfalls since inception and report the market cap of the funds in order to benchmark the numbers to the size of the fund. There is a large distribution of shortfalls between the different funds. The shortfalls depend on the index characteristics (especially its volatility and its path) and on the trading patterns of the investors (the time they held it and the correlation between their trading and the underlying index performance). It is important to notice that on average, the investor experienced a financial shortfall on aggregate in all these five ETFs. Due to the path dependence nature of leveraged and inverse ETFs a shortfall is not always guaranteed for every investor compared to investing in a margin account, but the fact that we observe these shortfalls illustrates the likely misunderstanding of investors about the nature of

these investments and the unique risks that their daily rebalancing generate for longer term investing.

Table 5: Calculated holding periods of five leveraged ETFs.

ETF	Leverage Ratio	Inception	Market Capitalization at Inception	Estimated Aggregate Shortfall
DPK	-3	12/17/2008	$ 6,046,000	$ 1,412,489
TYO	-3	4/15/2009	$ 6,100,000	$ 745,502
RHO	-2	6/12/2008	$ 7,802,000	$ 207,726
SBB	-1	1/25/2007	$ 15,630,750	$ 1,573,060
UVG	2	2/22/2007	$ 10,407,000	$ 464,699

IV. Investors' Investment Horizon and Funds' Compounding Frequency

The losses suffered by investors in leveraged and inverse ETFs has lead FINRA to issue regulatory notices. Moreover, new offerings by Direxion rebalance their portfolios only once a month. These new investments claim to match better the investment horizon of investors. However, given that investors' investment horizon varies across investor, it is important to understand whether these new investments (or similar ones) add value for long-term investors in leveraged investments. Little (2010) explains the concepts behind these investments.

In Table 6, we calculate the average difference between an investment in the benchmark portfolio and an investment in equivalent leveraged and inverse ETFs. The leveraged ETF will compound daily while the benchmark portfolio rebalances only at the end of the holding periods. Table 6 reports the difference between the two strategies. Since the compounding effect depends on the volatility of the underlying index we report the results for four different indexes with differing volatilities, and we present our results both for leveraged and inverse leveraged ETFs. In order to calculate the returns we randomly sample dates in the life of the funds and hold the investment for a week, a month, or a quarter and compare the performance of the different strategies.

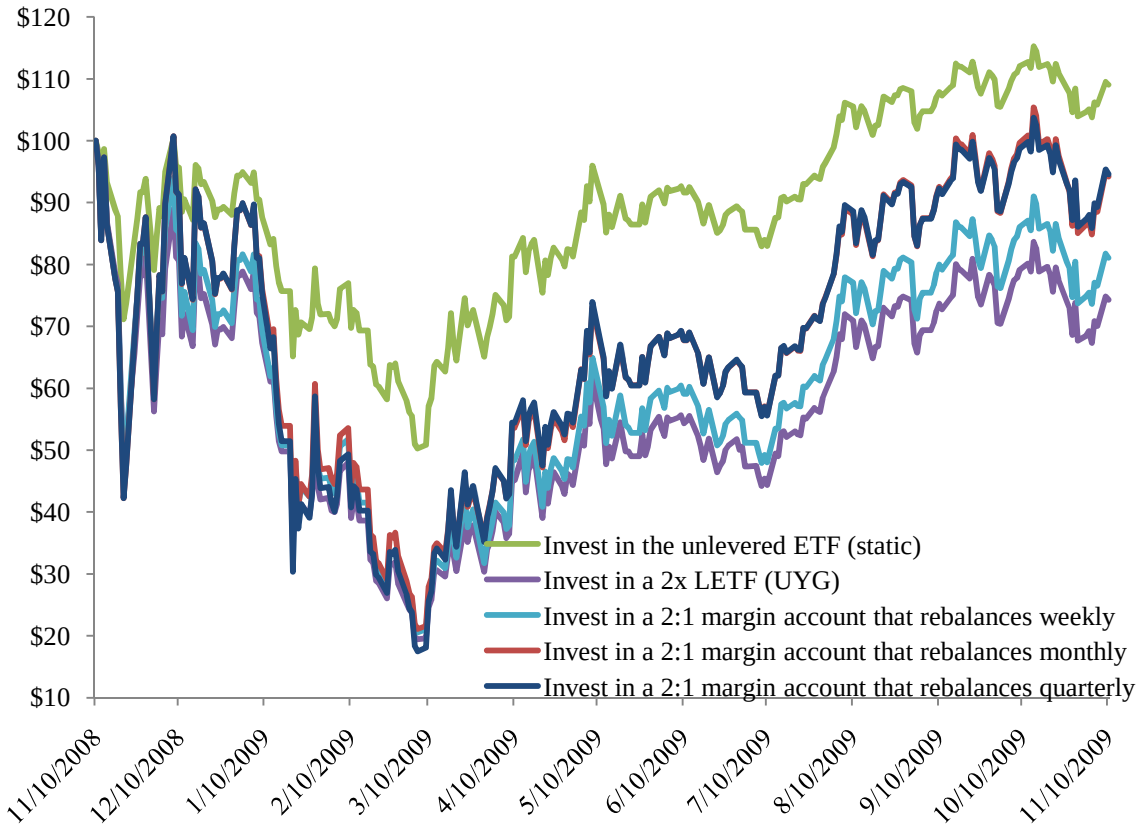
Table 6: Performance Slippage and Rebalancing Frequency

ETF	Holding Period		
	Weekly	Monthly	Quarterly
DJUSFN - Dow Jones U.S. Financials Index			
2x Leveraged (UYG)	-0.14%	-1.03%	-1.99%

(-2x) Inverse Leveraged (SKF)	-0.44%	-3.81%	-13.63%
Annualized Volatility of index	43.9%	40.3%	40.8%
DJUSUT - Dow Jones U.S. Utilities Index			
2x Leveraged (UPW)	-0.04%	-0.26%	-0.63%
(-2x) Inverse Leveraged (SDP)	-0.57%	-2.90%	-9.42%
Annualized Volatility of index	21.5%	21.7%	21.8%
RGUSFL - Russell 1000 Financial Services			
3x Leveraged (FAS)	-0.44%	-4.02%	-10.70%
(-3x) Inverse Leveraged (FAZ)	-1.13%	-7.28%	-15.36%
Annualized Volatility of index	48.4%	44.0%	42.8%
RIY - Russell 1000			
3x Leveraged (BGU)	-0.22%	-1.34%	-3.78%
(-3x) Inverse Leveraged (BGZ)	-0.34%	-1.87%	-4.59%
Annualized Volatility of index	25.3%	22.7%	20.3%

Table 6 shows that the mismatch between the investor's horizon and the daily horizon of an ETF cause on average a large difference in performance. For example, if an investor plans to hold twice the inverse of the Dow Jones U.S. Financial Index (DJUSUT) they could do so in a variety of ways. If they held the inverse leveraged ETF -- SKF, over 2007-2009, they would lose on average 0.44% compared to the benchmark. If their horizon were a month, they would lose 3.81% in that month compared to the benchmark. If the investor's horizon were a quarter, they would have lost 13.63% compared to the benchmark.

Figure 5: Comparison of holding returns using different compounding periods



As Table 6 and Figure 5 show, a rebalancing schedule of a month or longer yields a much more similar return to a benchmark portfolio, which does not rebalance at all. This highlights two important facts. First, investments with different rebalancing schedules may add value to investors, as they may be more suited to their needs. Second, the optimal way for investors to properly invest in a leveraged investment is by generating the leverage themselves using a margin account. Though investments with longer rebalancing periods may be better on average, they do not deliver the exact expected return as an investment in a margin account.

If an ETF offers a monthly rebalancing schedule, any investor buying such an investment in the middle of the month will invest in a vehicle that does not have exactly the stated leverage. However, this is no different a problem that an investor faces with current ETF and the fact that the investor needs to daily rebalance their portfolio if they want to continue match their horizon using a daily rebalancing horizon vehicle (dynamic rebalancing).

Using our MTM methodology in order to calculate holding periods using trading volume data, we calculate the estimated shortfall from an investment in a theoretical ETF that rebalances monthly. The results of our calculations are presented in Table 7.

Table 7: Investment Shortfalls calculated if investors had invested in a leveraged fund that rebalanced only monthly

Name	Leverage Ratio	Inception Date	Shortfall of Margin Account – Leveraged ETF	Theoretical Shortfall of Margin Account – Monthly Rebalanced ETF
DPK Equity	-3	12/17/2008	$ 1,412,489	$ 78,526
TYO Equity	-3	4/15/2009	$ 535,768	$ (311,017)
RHO Equity	-2	6/12/2008	$ 207,726	$ (12,389)
SBB Equity	-1	1/25/2007	$ 1,573,060	$ (863,744)
UVG Equity	2	2/22/2007	$ 464,699	$ 968,306

The results are surprising. Similar to the evidence implied by Table 6 and Figure 5, the actual shortfall is smaller for a monthly rebalancing ETF for most but not all ETFs. Moreover, the relationship between the two shortfalls is not always as strong or in the same direction. DPK's shortfall nearly disappears with a monthly rebalancing, while UVG's shortfall nearly doubles. TYO, RHO, and SBB not only see their shortfall shrink but it becomes negative. This means that for these three ETFs an investor with the same investment patterns would have made a higher return using a monthly rebalancing fund than a margin account. The reason for these substantial discrepancies is that monthly-rebalancing ETFs' leverage changes daily, as the ETF does not rebalance daily. Clearly, a monthly rebalancing portfolio may be better for longer horizon investors but it is far from being a simple or perfect solution. The nature of leveraged or inverse ETFs that they need to rebalance daily (or even monthly) add an element of risk to the investment compared to a margin account, which is not clearly fully understood by the investors.

V. Conclusions

Cheng and Madhavan (2009) and Little (2010) argue that leveraged and inverse ETFs do not deliver the expected return investors may expect when they invest in them for periods longer than a day or two. FINRA has required the issuers of these ETFs to

caution their customers that these should be short-term investments and need to be monitored carefully.

In this paper, we follow this argument and investigate it further by estimating from the data what is the distribution of the investors' holding periods in those ETFs. We find that many investors hold their leveraged ETFs for very long periods, at times longer than 3 month. Further, we calculate the financial shortfall of such a behavior compared to investing and creating the leverage in a margin account. We find that some of these ETF investors lose a substantial amount of money, up to 3% of their original investment in 3 weeks, the equivalent of 50% annualized. This indicates that the investors do not fully comprehend the risks associated with using a short-term investment vehicle as a long-term investment.

Further, we investigate the value of ETFs that rebalance monthly instead of daily in order to match better the need of their investors. We find that indeed the shortfall is smaller but not absent. Moreover, we find that such instruments tend to have returns that are more similar to investing in a margin account, but they may have added risk as the investor does not get exactly the leverage they sought.

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VII. Appendix I - Multiple Trading Models in Assessing the Investors' Holding Period and Shortfall When Investing in Leveraged and Inverse Leveraged ETFs

We follow the standard methodology as explained by Barclay and Torchio (2001), McCann and Hsu (1999), and Beaver, Malernee and Keeley (1997). In this appendix, we describe the steps we follow.

Assumptions:

(1) 30% of the daily trading volume comes from the specialists trading.

(2) We assume shorting is zero
(3) We do not include large block creation or redemption as part of the trading volume as it is not done on the secondary market.
(4) Use MTM model. High activity traders trade 80% of the time and hold 20% of the shares; low activity traders trade 20% of the time and hold 80% of the shares.

Procedures:

All total shares bought and sold are adjusted by specialists trading first. On day 0, the initial total shares outstanding equals to the number of shares bought on day 0, no sales occurred. Assume H0 is already adjusted by specialists' trading. The total shares outstanding are bought by two groups of investors – the high activity trader and the low activity trader. The high activity traders hold 20% of the shares but they trade 80% of the time. The low activity traders hold 80% of the shares but they trade 20% of the time. Let H0 denote the number of shares that high (low) activity traders hold at day 0. For example, if the total shares outstanding on day 0 was 1,000,000 shares, high activity traders hold 20%, i.e., H0 is equal to 200,000 shares; while low activity traders hold 80%, i.e., H0 is equal to 800,000 shares.

$$B_0 = H_0, S_0 = 0$$

The buy volume Bt or sell volume St is divided into 80% traded by high activity traders and 20% traded by low activity traders.

Next, we do the calculations for each group separately. For high (low) activity traders, we compute the sales rate as a percentage of group sales volume over previous day's total shares outstanding for that group, i.e.

$$R_t = \frac{S_t}{H_{t-1}}$$

On day 1, the number of shares from day 0 that get sold is B0 * R1, at price P1. The number of shares bought on day 0 that are retained is B0 * (1- R1), at price P0. The number of shares that are newly purchased is B1, at price P1. Holding period for B0*R1 shares is 1 day.

On day 2, the number of shares from day 0 that get sold at price P2 is B0*(1-R1)*R2– hence the holding period for these shares is 2 days; the number of shares

bought on day 1 that get sold is B1*R2, at price P2, with 1 day holding period. The number of shares from day 0 that are retained is B0*(1-R1)*(1-R2); the number of shares from day 1 that are retained is B1*(1-R1)*(1-R2). The algorithm also ensures that the total number of sells on day 2 equals B0*(1-R1)*R2 + B1*R2.

We repeat this process for each trading day purchases and sales, based on the real data of trading volume. Over time, the shares bought on the first trading day will get sold proportionally to the sales rate on each of the following trading day. Table 1 is the transaction matrix that shows the number of shares that are sold each trading day (the first column) that matches the original purchase dates (the first row). For example, B0 * R1 is the total number of shares that were sold on day 1 but comes from the purchase of day 0, resulting a 1-day holding period; B0*(1-R1)*R2 is the total number of shares that were sold on day 2 but comes from the purchase of day 0, resulting in a 2-day holding period; and the B1*R2 is the number of shares that were sold on day 2 but comes from the purchase of day 1, resulting in a 1-day holding period, and so on.

Table 8: The number of sold shares matched with the original purchase dates

Sell\Buy Date	**Day 0**	**Day 1**	**Day 2**	**……**
Day 1	$B_0 * R_1$			
Day 2	$B_0*(1-R_1)*R_2$	B_1*R_2		
Day 3	$B_0*(1-R_1)*(1-R_2)*R_3$	$B_1*(1-R_2)*R_3$	B_2*R_3	
……				

The transaction matrix summarizes to the distribution of holding period for all the shares that were bought over the entire trading period. We plot the cumulative distribution of the shares (%) that were held for 1 day, 2 days, 3 days, etc., until 99% of the shares were sold.

The holding period return difference between investing in LETF vs. ETF, LETF vs. Margin account ETF and LETF vs. Treasury Fund, for each holding period is plotted. For example, we compute the average holding period return for all shares that were held for one trading day for leveraged/inverse ETFs and compare with that of a margin account over same holding period.

Next, we compute the total dollar amount of all shares that were sold for one-day holding period for leveraged/inverse ETFs and compare with the total dollar amount of all shares that were sold for one-day holding period for a margin account, or for an unlevered ETF (or the underlying index), or for treasury index. For each holding period, we compute the total amount of shortfall/profits and plot the distribution.